

Second Semester B.E. Degree Examination, Aug./Sept.2020 Engineering Mathematics – II

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Solve: $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 8y = 0$ (06 Marks)
- b. Solve: $y'' - 4y' + 13y = \cos 2x$ (07 Marks)
- c. Solve by the method of undetermined coefficients of the equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^x$ (07 Marks)

OR

- 2 a. Solve: $\frac{d^2y}{dx^2} - 4y = \cosh(2x - 1)$ (06 Marks)
- b. Solve: $y'' + 3y' + 2y = 12x^2$ (07 Marks)
- c. Solve by the method of variation of parameters: $y'' + a^2y = \sec ax$ (07 Marks)

Module-2

- 3 a. Solve: $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = (1 + x)$ (06 Marks)
- b. Solve: $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$ (07 Marks)
- c. Obtain the general solution and the singular solution of the equation $x^4p^2 + 2x^3py - 4 = 0$ (07 Marks)

OR

- 4 a. Solve: $(1 + x)^2\frac{d^2y}{dx^2} + (1 + x)\frac{dy}{dx} + y = \sin 2[\log(x + 1)]$ (06 Marks)
- b. Solve: $p^2 + 2py \cot x = y^2$ (07 Marks)
- c. Solve the equation $(px - y)(py + x) = 2p$ by reducing into Clairaut's form, taking the substitutions $X = x^2, Y = y^2$. (07 Marks)

Module-3

- 5 a. If $z = e^{ax+by} f(ax - by)$, then show that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$. (06 Marks)
- b. Solve $\frac{\partial^2 z}{\partial x^2} = xy$ subject to the conditions that $\frac{\partial z}{\partial x} = \log(1 + y)$ when $x = 1$, and $z = 0$ when $x = 0$. (07 Marks)
- c. Find the solution of one dimensional wave equation, using the method of separation of variables. (07 Marks)

OR

- 6 a. Form the PDE by eliminating the arbitrary functions ϕ from $lx + my + nz = \phi(x^2 + y^2 + z^2)$ (06 Marks)
- b. Solve $\left[\frac{\partial^2 z}{\partial x \partial y} \right] = \sin x \sin y$ subject to the condition $\left[\frac{\partial z}{\partial y} \right] = -2 \sin y$ when $x = 0$ and $z = 0$ if y is a odd multiple of $\pi/2$. (07 Marks)
- c. Derive an one dimensional heat equation in the form $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$. (07 Marks)

Module-4

- 7 a. Change the order of integration and hence evaluate $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} xy \, dy \, dx$ (06 Marks)
- b. Evaluate $\int_0^a \int_0^{x+y} \int_0^{x+y+z} x + y + z \, dz \, dy \, dx$ (07 Marks)
- c. Show that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ with usual notations. (07 Marks)

OR

- 8 a. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} \, dx \, dy$ by changing to polar coordinates. (06 Marks)
- b. Find by double integration the area enclosed by the curve $r = a(1 + \cos\theta)$ between $\theta = 0$ and $\theta = \pi$. (07 Marks)
- c. Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. (07 Marks)

Module-5

- 9 a. Find the Laplace transform of $f(t) = t(\sin t)$. (06 Marks)
- b. If $f(t) = t^2$, $0 < t < 2$ and $f(t+2) = f(t)$ for $t > 2$, find $L\{f(t)\}$. (07 Marks)
- c. Find the inverse Laplace transform of $\left[\frac{s+5}{s^2-6s+13} \right]$. (07 Marks)

OR

- 10 a. Find the Laplace transform of the unit step function $f(t) = \begin{cases} \sin t, & 0 < t \leq \frac{\pi}{2} \\ \cos t, & t > \frac{\pi}{2} \end{cases}$ (06 Marks)
- b. Employ Laplace transform to solve the equation $y'' + 5y' + 6y = 5e^{2x}$ (07 Marks)
- c. Find the inverse Laplace transform of $\frac{s^2}{(s^2+a^2)^2}$, using convolution theorem. (07 Marks)
